

Tips & Techniques

Step by Step through Calculus I Problems

One of the revolutionary features in Maple® 8 is the ability to walk or single step through calculus problems. With the ability to interactively apply rules to limit, differentiation and integration problems, students can drill themselves on calculus rules and help ingrain these concepts. Not only can you apply rules to a problem, you can also ask Maple for a hint if you get stuck during any stage in stepping through a problem. If you already understand one or more rules in limit, differentiation or integration problems, this package allows you to step over and avoid any understood rules.

Like all packages in Maple, the package needs to be loaded before accessing any of the routines. Load this package in a Maple worksheet by typing:

```
> with(Student[Calculus1]);
```

Several of the commands in the **Calculus1** package will deliver helpful information such as stating the problem number and providing detailed information to assist you while stepping through a problem. In order to see these messages, the `infolevel` needs to be set for either the **Student** package or the **Calculus1** subpackage.

```
> infolevel[Calculus1] := 1;
```

In order to take advantage of the step-into functionality, problems need to be represented using the inert Maple forms: `Limit`, `Diff`, and `Int`.

```
> Diff(sin(x^3)/x, x);
```

$$\frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right)$$

The `Rule` command lets you apply a rule to the problem where the rule name is stated in square brackets and the problem, to which this rule will be applied, in parenthesis.

The first step in solving this problem requires the `quotient` rule.

```
> Rule[quotient](Diff(sin(x^3)/x, x));
```

Creating problem #1

$$\frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right) = \frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3) \left(\frac{d}{dx} x \right)}{x^2}$$

The output from a call to `Rule` is normally an equation whose left hand side is the original problem and whose right hand side is the current state of that problem. This output can be passed to another application of the `Rule` command.

The next step in solving this problem is to apply the identity rule to

$$\frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3) \left(\frac{d}{dx} x \right)}{x^2}$$

the current state of the problem,

```
> Rule[identity]( (Diff(sin(x^3),x)*x-
sin(x^3)*Diff(x,x))/x^2 );
```

$$\frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right) = \frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3)}{x^2}$$

During any stage in solving a problem, if you are uncertain which rule is required, you can ask Maple for a hint. In this case, we want to receive a hint for

$$\frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3)}{x^2}$$

```
> Hint((Diff(sin(x^3),x)*x-sin(x^3))/x^2);
```

```
[chain]
```

The `Hint` command states that the `chain` rule is the next step in solving this problem.

```
> Rule[chain]((Diff(sin(x^3),x)*x-sin(x^3))/x^2);
```

$$\frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3)}{x^2} = \frac{\left(\frac{d}{dx} \sin(_X) \right) \Big|_{x=x^3} x^2 - \sin(x^3)}{x^2} \left(\frac{d}{dx} (x^3) \right) x - \sin(x^3)$$

We now see that the `power` rule is required. Instead of writing the problem definition every time a rule is to be applied, we can take advantage of the ditto operator (`%`) in Maple. The ditto operator (`%`) refers to the output of the last command executed. This will allow us to easily apply the `power` rule to this problem and avoid superfluous typing.

```
> Rule[power](%);
```

$$\frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right) = \frac{3 \left(\frac{d}{dx} \sin(_X) \right) \Big|_{x=x^3} x^2 - \sin(x^3)}{x^2}$$

The final step in solving this problem requires the `sin` rule.

```
> Rule[sin](%);
```

$$\frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right) = \frac{3 \cos(x^3) x^2 - \sin(x^3)}{x^2}$$

To see the steps taken from the beginning of a problem to its current state, use the `ShowSteps` command.

```
> ShowSteps();
```

The **Calculus1** package also contains numerous routines to help visualise concepts presented in a Calculus course such as `FunctionChart`, `IntegrationPlot`, and `DerivativePlot`. For more information on this package, enter `?Student[Calculus1]` in

$$\begin{aligned} \frac{d}{dx} \left(\frac{\sin(x^3)}{x} \right) &= \frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3) \left(\frac{d}{dx} x \right)}{x^2} \\ &= \frac{\left(\frac{d}{dx} \sin(x^3) \right) x - \sin(x^3)}{x^2} \\ &= \frac{\left(\frac{d}{dx} \sin(_X) \right) \Big|_{x=x^3} x^2 - \sin(x^3)}{x^2} \left(\frac{d}{dx} (x^3) \right) x - \sin(x^3)}{x^2} \\ &= \frac{3 \left(\frac{d}{dx} \sin(_X) \right) \Big|_{x=x^3} x^2 - \sin(x^3)}{x^2} \\ &= \frac{3 \cos(x^3) x^2 - \sin(x^3)}{x^2} \end{aligned}$$

the Maple prompt or visit the Maple Application Center to view the Single-Stepping through Calculus Problems in Maple 8 demo worksheet at <http://www.adeptscience.com/go?pg=076>

